Desafios na Simulação Numérica de Equações Diferenciais Parciais - III: Métodos de Galerkin Descontínuos, Mistos e Híbridos em Problemas da Engenharia*

Abstract

Com o aumento da capacidade computacional, tem se tornado cada vez mais viável e atrativo o uso de formulações de elementos finitos que forneçam aproximações confiáveis para múltiplas quantidades de interesse associadas a um fenômeno físico, o que abre caminho para o uso de formulações descontínuas, mistas e/ou híbridas nas simulações de problemas da Engenharia. O desenvolvimento dessas formulações almeja estudar com maior exatidão problemas complexos envolvendo duas ou mais variáveis.

O objetivo do presente minissimpósio é reunir pesquisadores brasileiros, com alguns convidados estrangeiros, que trabalhem nesta área. Espera-se proporcionar uma discussão sobre os recentes esforços dos participantes, seus projetos, suas dificuldades, os desafios decorrentes da resolução de problemas da Engenharia, possibilidades de novas colaborações e, principalmente, despertar interesse em jovens pesquisadores, a maioria dos participantes do CNMAC.

As palestras serão principalmente em áreas de aplicação, como modelagem de escoamentos multifásicos no subsolo, problemas multifísicos, entre outros.

Organizadores

- Abimael Loula - LNCC, RJ
- Cristiane Faria - UERJ, RJ
- Omar Durán, LabMeC-FEC, Unicamp, SP
- Philippe Devloo - FEC -Unicamp, SP
- Thiago Quinelato, LabMeC-FEC, Unicamp, SP

Proposta de programa

As palestras estão organizadas em duas sessões de quatro palestrantes cada. A seguir segue a divisão e os títulos e resumos de cada uma.

- 1^a parte:
    1. **Integrating geometric modelling with numerical simulation: the scaled boundary finite element approach**, Chongmin Song, University of New South Wales, Australia
    2. **Desafios na implementação de métodos mistos e híbridos: aspectos de estruturação e paralelização de código**, Antonio Tadeu de Azevedo Gomes, LNCC, RJ
    3. **Stabilized Hybrid Descontinuous Galerkin Methods for the Heat Problem**, Cristiane Faria - UERJ, RJ
    4. **On a Multiscale Approximation of Darcy Flow with Fracture Networks**, Philippe R. B. Devloo - FEC, Unicamp
- 2^a parte:
    1. **Estimativas a posteriori de erro em métodos de Galerkin descontinuo no tempo e espaço para aproximação de problemas parabólicos**, Igor Mozolevski - UFSC, SC

*As duas versões anteriores deste minissimpósio ocorreram no CNMAC 2010 e 2016

2. **A New Sequential Method for Three-Phase Immiscible Flow in Poroelastic Media**, Marcio Murad - LNCC, RJ
3. **Multiscale Hybrid-Mixed Methods for Fluids**, Frédéric Valentin - LNCC, RJ
4. **Enriched Approximation Space Configurations for Mixed Finite Element Methods: Darcy's, Stokes and Elasticity Problems**, Sônia Gomes - IMMEC, Unicamp

Resumos

1. **Desafios na implementação de métodos mistos e híbridos: aspectos de estruturação e paralelização de código**

   Antônio Tadeu de Azevedo Gomes - LNCC/MCTI

   Nesta palestra discorremos sobre a experiência do grupo de pesquisa MHM (Multiscale Hybrid Mixed) do LNCC no desenvolvimento de simuladores numéricos paralelos baseados em uma família de métodos numéricos multiescala híbridos mistos desenvolvida pelo grupo. São apresentados dois enfoques, um na estruturação do código considerando abstrações orientadas a objetos e a design patterns as mais próximas possíveis de seus equivalentes na família de métodos numéricos estudada, e outro na exploração de diferentes níveis de paralelismo (intra-nó, inter-nó, atores, aceleradores) induzidos pela concepção matemática dessa família de métodos. O resultado deste trabalho é uma biblioteca C++ denominada MSL (MHM Software Library), que hoje se aproxima das 100.000 linhas de código, com suporte a MPI e OpenMP, integrada a diferentes bibliotecas de alto desempenho como Pardiso, MUMPS e PaStiX, e que vem sendo instanciada para uma gama de diferentes problemas.

2. **Integrating geometric modelling with numerical simulation: the scaled boundary finite element approach**

   Chongmin Song
   Centre for Infrastructure Safety and Engineering
   School of Civil & Environmental Engineering,
   University of New South Wales, Sydney, NSW 2052, Australia

   Finite element analysis has transformed the way engineering analyses and designs are performed in practice. Currently, finite element analysis is too often still a time-consuming, expensive and daunting task in engineering, largely due to the human effort required to generate finite element models. Furthermore, digital images and visual reality models are increasingly popular in engineering, which poses additional challenges to the conventional finite element method.

   In this presentation, our recent research on integrate geometric model with numerical simulation will be introduced. This research aims to save real time and money in engineering analysis and design. Underpinning this effort is the scaled boundary finite element method that we have developed. This method allows the seamless integration of geometric models (in CAD, digital image and VR formats) and finite element analysis. Human interventions are greatly reduced in the analysis process. Potential applications, developments and benefits will be discussed with proof-of-concept type of examples.

3. **A New Sequential Method for Three-Phase Immiscible Flow in Poroelastic Media**

   Marcio Murad - LNCC/MCTI

   A new computational model is developed to solve three phase immiscible compressible flow in strongly heterogeneous poroelastic media. Within the context of the iterative coupled formulation based on the fixed stress split algorithm the governing equations are decomposed into three subsystems associated with the geomechanics, hydrodynamics and transport problems. The hydrodynamics involves an overall compressibility which plays essential role in the magnitude of additional source in the pressure equation associated the time derivative of the total mean stress. We show that the additional coefficients in the source can be decomposed into a rock/grain and fluid components involving the formation volume factors of each phase. The flow equations are written in a proper conservative form for application of mixed-hybrid formulation whereas the non-linear coupling between flow and transport is handled by a proper sequential iterative scheme. A new post-processing approach is proposed to update the Lagrangian porosity based on the freezing of the total mean stress which reproduces the one-way formulation in a straightforward fashion under the stationarity of the total mean stress. The geomechanical subsystem is also discretized by a mixed formulation based on the decomposition of the effective stress into spherical and deviatoric components in order to handle the nearly-incompressible undrained limit. The system of conservation laws for the liquid saturations is rephrased in an alternative form for an appropriate operator splitting method, leading to the appearance of an extra source term arising from the transient behavior of the Lagrangian porosity induced by the geomechanical coupling. Within the framework of a predictor-corrector scheme, the predictor step is discretized by an extended version of a higher order central-upwind scheme whereas the corrector, which captures the influence of rock skeleton deformation upon transport, is captured by a Godunov splitting. The potential of the new computational model is illustrated in numerical simulations of water-flooding and of WAG (Water Alternating Gas) injection problems in poroelastic media. Numerical results illustrate precisely the role of the transient nature of the total stress upon subsidence, rock compaction, oil and gas production and finger grow in primary and secondary recovery regimes.

4. **On a Multiscale Approximation of Darcy Flow with Fracture Networks**

Philippe Devloo, FEC-Unicamp

In the numerical simulation of fluid flow through a fracture network, a potencially very large number of fractures create a flow pattern in a lower dimensional domain embedded in the fluid flow modeled by the Darcy equations. In order to accurately model this coupled model between fracture flow and volumetric flow, very fine meshes need to created to capture the essential features of the fluid flow. In this work we develop a mathematical mixed formulation allowing to couple the volumetric with the fracture flow while maintaining the features of local conservation. This formulation is then extended to allow a multiscale approach where the fracture network are separated in macro domains that interact with each other through macroscopic interfaces. The resulting approximation technique allows to simulate fracture networks accurately with a reduced number of global system equations. The technique is implemented in the NeoPZ finite element environment and its effectivity demonstrated numerically.

5. **Estimativas a posteriori de erro em métodos de Galerkin descontinuo no tempo e espaço para aproximação de problemas parabólicos**

Igor Mozolevski - UFSC, SC

Para problemas de advecção-difusão transientes consideram-se estimativas de erro em métodos de Galerkin descontinuo no tempo e espaço. Estimativas de erro na norma de energia e em aproximação de funcional de interesse são desenvolvidos usando técnicas de fluxos equilibrados. Indicadores de erro baseados nestas estimativas são aplicados para adaptação dinâmica de malha no tempo e espaço em aproximação de problemas de advecção-difusão.

6. **Stabilized Hybrid Descontinuous Galerkin Methods for the Heat Problem**

Cristiane Faria, IME, UERJ

Higher-order methods combining a stabilized hybrid method for the spatial variable with linear multistep methods for the time variable are presented and analyzed for the heat problem. The hybrid method, originally introduzed for elliptic problems [1], consisting of coupling local problems resolved by the discontinuous Galerkin method (DG) for the primal variable with a global problem for the Lagrange multiplier, which is identified with the temperature trace, weakly imposing continuity between the elements. The proposed hybrid formulations preserve the main properties of the associate DG methods such as consistency, stability, boundedness and optimal rates of convergence in the energy norm; however, with reduced complexity and computational cost. A stability analysis of the full discrete problem is presented when the Crank-Nicolson scheme is used in the time aproximation. Numerical results of convergence studies confirm the rates of convergence predicted by the numerical analysis.

7. **Multiscale Hybrid-Mixed Methods for Fluids**

Frédéric Valentin, LNCC/MCTI

This work presents a family of finite element methods for multiscale fluid problems, named Multiscale Hybrid-Mixed (MHM) methods. The MHM method is a consequence of a hybridization procedure which characterizes the unknowns as a direct sum of a "coarse" solution and the solutions to problems with Neumann boundary conditions driven by the multipliers. As a result, the MHM method becomes a strategy that naturally incorporates multiple scales through multiscale basis functions while providing solutions with high-order precision for the primal and dual variables. The completely independent local problems are embedded in the upscaling procedure, and then, computational approximations may be naturally obtained in a parallel computing environment. The numerical analysis for the one- and two-level versions of the MHM method shows that the method is optimal convergent and achieves super-convergence for the locally conservative velocity field. A face-based a posteriori estimator is proposed which is locally efficient and reliable with respect to the natural norms. The general framework and some recent results are illustrated for the Darcy, Stokes and Brinkman equations and validated through a large variety of numerical results for highly heterogeneous coefficient problems.

8. **Enriched Approximation Space Configurations for Mixed Finite Element Methods: Darcy's, Stokes and Elasticity Problems**

Sônia M. Gomes * & Philippe R. B. Devloo & Thiago Quinelato & Shudan Tian
* *IMECC–Universidade Estadual de Campinas, SP, Brazil)*

The purpose of the talk is to give an overview on recent research results pursuing enhanced approximate solutions of the mixed finite element method for these three kinds of equations. The current focus is on the mixed formulations for linear elasticity problems with weak stress symmetry. The adopted approximations are based on the application of enriched versions of classic stable spaces for Darcy's problems, for stress and displacement variables, and/or on enriched Stokes-compatible space configurations, for the choice of multiplier spaces used to weakly enforce stress symmetry. Accordingly, the stress space has to be adapted to restore stability. Such enrichment procedures are based on incrementing the spaces with extra bubble terms, a technique widely employed in the stabilization and/or enhancement of numerical methods for partial differential equations. Bubbles refer to functions with support in a single element (in the case of H^1-conforming approximations) or with vanishing normal components over element edges (in the case of $\mathbf{H}(\text{div})$-conforming vector spaces). Another advantage of using bubbles for methods improvement relies on the fact that their corresponding degrees of freedom can be condensed without affecting the number of equations to be solved, and the matrix structure. An error analysis shows that using these kind of enriched space configurations for elasticity problems can produce accuracy enhancement of the divergence and displacement variables and/or accuracy improvement of the weak stress symmetry enforcement, but stress L^2-errors are not affected. Some numerical tests shall be presented for the verification of the predicted error estimates.