Minissimpósio "Métodos Computacionais de Otimização"

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1 Objetivos

O minissimpósio em Métodos Computacionais de Otimização tem como objetivo divulgar para a comunidade de matemática aplicada a pesquisa em alto nível desenvolvida por pesquisadores e pesquisadoras da área de otimização contínua no Brasil. Além disso, pretende-se que o minissimpósio propicie um ambiente de discussão entre participantes de diferentes regiões do país com o intuito de fomentar novas colaborações.

2 Descrição

A área de Otimização Contínua estuda o problema de minimizar uma função real a várias variáveis reais (função objetivo) dadas restrições sob as variáveis de entrada. Isto é,

$$\text{Minimizar } f(x), \text{ sujeito a } x \in \Omega,$$

onde $f\colon \mathbb{R}^n \to \mathbb{R}$ e $\Omega \subseteq \mathbb{R}^n$. Tipicamente o conjunto viável Ω é descrito por restrições de igualdade e desigualdade. Um número incontável de problemas práticos é modelado desta forma, desde problemas de energia, de economia, de engenharia, química, etc, até problemas nas áreas de ciências sociais. Particularmente com o avanço das ferramentes computacionais, modelos mais realistas e que portanto fazem uso de toda a generalidade acima (isto é, sem hipóteses de linearidade ou de convexidade sob as funções que definem o problema) tem sido propostos. Um exemplo de destacado sucesso são as aplicações pioneiros de técnicas de aprendizado de máquina que necessitam a resolução de problemas de otimização complexos e não convexos (em particular, técnicas de *deep learning*). Neste minissimpósio trazemos 14 palestrantes de diferentes regiões do país, a maioria deles pesquisadores consolidados em suas respectivas áreas, para palestras de 15 a 20 minutos, totalizando 4h de duração.

3 Programação

Horário	**5a feira 18/09**	**Horário**	**6a feira 19/09**
14:30 - 14:50	Leandro Prudente	8:00 - 8:20	Vitaliano Amaral
14:50 - 15:10	Roberto Andreani	8:20 - 8:40	Orizon Ferreira
15:10 - 15:30	L. Rafael Santos	8:40 - 9:00	Mariana da Rosa
15:30 - 15:50	Evelin Krulikovski	9:00 - 9:20	Juan Pablo Luna
15:50 - 16:10	Marina Geremia	9:20 - 9:40	Diaulas Marcondes
16:10 - 16:30	Roger Behling	9:40 - 10:00	Gabriel Grillo
			Daiana Santos

4 Resumos

1. Daiana Oliveira dos Santos (UNIFESP)

Optimality Conditions for Nonlinear Second-Order Cone Programming and Symmetric Cone Programming

Nonlinear symmetric cone programming (NSCP) generalizes important optimization problems such as nonlinear programming, nonlinear semi-definite programming and nonlinear second-order cone programming (NSOCP). In this work, we present two new optimality conditions for NSCP without constraint qualifications, which implies the Karush–Kuhn–Tucker conditions under a condition weaker than Robinson's constraint qualification. In addition, we show the relationship of both optimality conditions in the context of NSOCP, where we also present an augmented Lagrangian method with global convergence to a KKT point under a condition weaker than Robinson's constraint qualification.

2. Luiz Rafael dos Santos (UFSC)

A successive centralized circumcentered-reflection method for the convex feasibility problem

In this paper, we present a successive centralization process for the circumcentered-reflection method with several control sequences for solving the convex feasibility problem in Euclidean space. Assuming that a standard error bound holds, we prove the linear convergence of the method with the most violated constraint control sequence. Moreover, under additional smoothness assumptions on the target sets, we establish the superlinear convergence. Numerical experiments confirm the efficiency of our method.

3. Diaulas S. Marcondes (USP)

Active-set Newton-MR methods for nonconvex optimization problems with bound constraints

Resumo: In this work we present active-set methods for the minimization of nonconvex twice continuously differentiable functions with bound constraints. Within the faces, descent methods with Armijo line search are employed with approximated Newton directions obtained by the Minimum Residual (MINRES) method. To leave the faces, we investigate the use of the Spectral Projected Gradient (SPG) method, and also the use of a variant of a Cubic Regularization of the Newton's method tailored to bound constrained problems. When the objective function has Lipschitz continuous gradient, we show that the method based on SPG takes needs no more than $\mathcal{O}(n\epsilon^{-2})$ calls to the oracle to find ϵ-approximate stationary points. If, additionally, the objective function has Lipschitz continuous Hessian, we show that the method based on the cubic regularization method needs no more than $\mathcal{O}\left(n\epsilon^{-3/2}\right)$ calls to the oracle to achieve the same goal. We also report numerical experiments comparing the proposed methods against existing active-set methods. The results illustrate the potential benefits of using MINRES instead of Conjugate Gradient (CG) method for approximating Newton directions within the faces.

4. Roger Behling (UFSC)

Computing the completely positive factorization via alternating minimization

In this article, we propose a novel alternating minimization scheme for finding completely positive factorizations. In each iteration, our method splits the original factorization problem into two optimization subproblems, the first one being a orthogonal procrustes problem, which is taken over the orthogoal group, and the second one over the set of entrywise positive matrices. We present both a convergence analysis of the method and favorable numerical results.

5. Juan Pablo Luna (UFRJ)

Modeling the New Brazilian Gas Market: Equilibrium Structure and Optimization Reformulation

Recent regulatory reforms in Brazil have significantly reshaped the natural gas sector, aiming to increase competition. These initiatives are part of a broader effort to attract new market participants and promote a more dynamic and efficient market structure. In this context, the Brazilian National Agency for Petroleum, Natural Gas and Biofuels (ANP) has outlined a strategic framework for the development of a competitive natural gas market, inspired by international

benchmark models. The agency's long-term vision is to transition Brazil from an emerging market to a mature one, characterized by liquid trading hubs and efficient price formation.

In this work, we present a mathematical analysis of a theoretical model of the restructured Brazilian natural gas market. The model incorporates multiple types of agents, including pre-salt gas producers, LNG importers, and various consumers such as refineries, power plants, and end-users. While the model reflects the complexity of the current logistical and distribution infrastructure -featuring key transportation operators— we abstract its mathematical core as a General Equilibrium Problem (GEP). Under suitable paradigm shifts and the introduction of additional mathematical structures, we show that the problem can be reformulated as a Generalized Nash Equilibrium Problem (GNEP). This equivalence offers new insights into the system's behavior and facilitates the use of advanced solution techniques, including reformulations as quadratic optimization problems in specific cases.

6. Evelin H. M. Krulikovski (UFPR)

   Derivative-free optimization approach for structured symmetric matrices with fixed eigenvalues

   A Derivative-Free Optimization (DFO) model is developed and analyzed for solving inverse structured symmetric matrix problems for which the eigenvalues are given. Some (zero and nonzero) entries are pre assigned and cannot be changed, and some others should be nonzero but their value is not given. The rest of the entries are completely free. The obtained matrix must satisfy these requirements and its eigenvalues must be the given ones. This specialized inverse eigenvalue problem appears in several applications and is related to the problem of determining the graph, with weights on the undirected edges, of the matrix associated with its sparse pattern. Our optimization model requires computing the eigenvalues of a symmetric matrix to evaluate the non-differentiable objective function. We propose derivative-free optimization schemes, specifically the well-known directional direct search method DDS and its global variant GLODS. We discuss their convergence properties which are based on the fact that the objective function is Lipschitz continuous. In addition, we also explore the potential benefits of using some well-established heuristic strategies that can be seen as natural competitors to the deterministic derivative-free schemes DDS and GLODS. We present some preliminary numerical results that demonstrate the effectiveness of our deterministic DFO proposals in a variety of possible scenarios.

7. Gabriel Grillo (USP)

   Uma aceleração do método de gradiente proximal baseada em inércia e aprendizado profundo

   O método de gradiente proximal é um pilar da Otimização Convexa, estendendo o método de gradiente descendente para o caso de função objetivo compósita, i.e., composta pela soma de duas funções convexas, sendo uma delas suave e a outra não suave. Por ser um método de primeira ordem, apropriado para problemas em alta dimensão, pesquisadores buscam por acelerações do método de gradiente proximal, visando melhores propriedades teóricas e práticas de convergência. Uma estratégia clássica de aceleração é a introdução de um termo de inércia. Nos últimos anos, o uso de estratégias de aprendizado profundo para aceleração de métodos de otimização vem ganhando destaque. Neste trabalho propomos um método com taxa de convergência $O(1/k^2)$ do valor funcional que utiliza inércia e aprendizado profundo para calcular o passo de aceleração. Experimentos numéricos ilustram o desempenho do método. Co-autores: Ernesto G. Birgin e Elias S. Helou.

8. Orizon P. Ferreira (UFG)

   Constraint Qualifications and Strong Global Convergence Properties of an Augmented Lagrangian Method on Riemannian Manifolds

   In the past several years, augmented Lagrangian methods have been successfully applied to several classes of nonconvex optimization problems, inspiring new developments in both theory and practice. In this paper we bring most of these recent developments from nonlinear programming to the context of optimization on Riemannian manifolds, including equality and inequality constraints. Many research have been conducted on optimization problems on manifolds, however

only recently the treatment of the constrained case has been considered. In this paper we propose to bridge this gap with respect to the most recent developments in nonlinear programming. In particular, we formulate several well-known constraint qualifications from the Euclidean context which are sufficient for guaranteeing global convergence of augmented Lagrangian methods, without requiring boundedness of the set of Lagrange multipliers. Convergence of the dual sequence can also be assured under a weak constraint qualification. The theory presented is based on so-called sequential optimality conditions, which is a powerful tool used in this context. The paper can also be read with the Euclidean context in mind, serving as a review of the most relevant constraint qualifications and global convergence theory of state-of-the-art augmented Lagrangian methods for nonlinear programming.

9. Vitaliano Amaral (UFPI)

   Um método do tipo ponto proximal com regularização quadratica para a minimização de uma composição de funções

   Discutiremos uma versão do método ponto proximal para resolver um problema de otimização composto pela soma de três funções: a primeira é uma função própria e semicontínua inferior, a segunda é continuamente diferenciável com gradiente Hölder contínuo, e a terceira é uma função convexa que pode ser não diferenciável. Esse tipo de problema surge em diversas aplicações práticas, mas é desafiador devido à presença de não convexidade e não suavidade. Para lidar com essas dificuldades, propomos um algoritmo baseado no método gradiente proximal, que combina uma aproximação quadrática do termo diferenciável, uma aproximação linear do termo convexo e um termo não convexo.

   O método proposto apresenta complexidade compatível com a dos métodos já estabelecidos na teoria da otimização. Além disso, apresentamos exemplos numéricos que ilustram os resultados teóricos e a eficiência prática do algoritmo.

10. Mariana da Rosa (Unicamp)

    Advances in constant-rank type constraint qualifications

    It is well known that constant rank-type constraint qualifications (CQs) imply the Mangasarian-Fromovitz CQ (MFCQ) after a suitable local reparametrization of the feasible set, which involves eliminating redundancies (remove and/or transform inequality constraints into equalities) without changing the feasible set locally. This technique has been mainly used to study the similarities between well-known CQs from the literature. In this talk, we present a different perspective by introducing a type of reparametrization that itself constitutes a CQ. Additionally, we propose a relaxed version of the constant rank of the subspace component (CRSC) CQ, called constrained CRSC, which preserves the relevant geometric properties of CRSC, is linked to reparametrizations to MFCQ, and also ensures local error bound condition.

11. Leandro Prudente (UFG)

    A Proximal Gradient method with an explicit line search for multiobjective optimization

    We present a proximal gradient method for solving convex multiobjective optimization problems, where each objective function is the sum of two convex functions, with one assumed to be continuously differentiable. The algorithm incorporates a backtracking line search procedure that requires solving only one proximal subproblem per iteration, and is exclusively applied to the differentiable part of the objective functions. Under mild assumptions, we show that the sequence generated by the method converges to a weakly Pareto optimal point of the problem. Additionally, we establish an iteration complexity bound by showing that the method finds an ε-approximate weakly Pareto point in at most $\mathcal{O}(1/\varepsilon)$ iterations.

12. Marina Geremia (IFSC)

    Um Algoritmo de Decomposição ADMM Inexato com Erro Relativo para Otimização Convexa com Efeitos Inerciais

    Este trabalho foi desenvolvido em colaboração com o Professor Maicon Marques Alves e propõe uma nova versão inexata com erro relativo do método dos multiplicadores de direção alternada

(ADMM) para otimização convexa. Demonstramos a convergência assintótica do nosso algoritmo principal, bem como as complexidades iterativas pontuais e ergódicas para os resíduos. Além disso, ilustramos a eficácia do algoritmo por meio de experimentos numéricos.

13. Roberto Andreani (Unicamp)

Um algoritmo geral de primeira ordem eficiente para encontrar a Região de Pareto

Do ponto de vista da teoria da decisão, o mais importante nos problemas de otimização é achar a região de Pareto. Para isso, é necessário primeiro encontrar pontos de Pareto. Existem vários algoritmos com maior ou menor eficiência para esse objetivo. Em geral, a região de Pareto é encontrada com estratégias de multistart.

Neste trabalho, propomos uma estratégia de coleta que generaliza os métodos de primeira ordem já propostos e que permite encontrar os pontos da região de Pareto ao longo do processo iterativo.

Mostramos resultados teóricos de convergência à região de Pareto da nova estratégia, bem como resultados numéricos que avaliam a eficiência da abordagem proposta.