PROGRAMAÇÃO MINISSIMPÓSIO - CNMAC 2024

Neste documento apresentamos uma proposta de minissimpósio em Teoria Espectral de Grafos para reunir os pesquisadores da área no CNMAC 2024.

Título: Teoria Espectral de Grafos

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1. Descriçao

Em Teoria Espectral de Grafos (TEG) associamos uma matriz M a um grafo G e estudamos como propriedades estruturais de grafos podem ser obtidas a partir das propriedades algébricas da matriz associada.

Em particular, dado que os autovalores e autovetores de M estão diretamente relacionados com invariantes de um grafo (distância média, diâmetro, raio, número isoperimétrico, energia, para mencionar alguns), eles podem fornecer informações úteis acerca do grafo ou acerca de uma aplicação modelada pelo grafo. Por exemplo, em muitos modelos de comunicação, o diâmetro desempenha um papel chave no desenho de redes. A energia de um grafo (soma dos valores absolutos dos autovalores da matriz de adjacência) é estudada intensamente em Química e pode ser usada para aproximar a energia total dos π-elétrons de uma molécula. A Teoria Espectral de Grafos também aparece naturalmente em vários problemas da Física Teórica e da Mecânica Quântica. Além disso, TEG mostra crescentes conexões com várias outras áreas, em especial com ciência de dados através dos algoritmos de Clusterização Espectral, e com otimização semidefinida. Esta última iremos explorar em detalhes com duas das palestras nesta proposta.

A TEG é uma área de pesquisa de muito prestígio internacional, com um crescente número de publicações em revistas reconhecidas. No Brasil ela está em franco estágio de desenvolvimento com pesquisadores reconhecidos internacionalmente. A comunidade é muito ativa e mostra sua importância na área, especialmente, em suas participações nas prestigiosas conferências internacionais do ILAS (International Linear Algebra Society).

2. Objetivos

Abaixo listamos os principais objetivos que pretendemos atingir com a realização da atividade:

- internacionalização dos grupos de pesquisa em TEG do Brazil;
- integração entre os vários grupos de pesquisa em TEG do Brazil;
- disseminar a pesquisa produzida pelos grupos de pesquisa e dessa maneira promover uma interação entre seus membros;
- captar novos estudantes para a área;

3. Indicação de conferencista

A indicação da Professora Aida Abiad como palestrante plenária no Congresso Nacional de Matemática Aplicada e Computacional é justificada por sua notável trajetória acadêmica e contribuições significativas para o campo, que a colocam como uma referência internacional, destacando-a como uma escolha ideal para compartilhar seus insights inovadores com a nossa comunidade.

A Professora Abiad possui uma extensa lista de publicações em revistas prestigiadas, o que atesta sua competência e relevância no cenário científico. Seus artigos têm impactado positivamente o avanço da teoria espectral dos grafos, demonstrando um profundo entendimento e domínio da área. Além disso, sua participação em projetos de colaboração com pesquisadores brasileiros fortalece ainda mais sua conexão com a comunidade acadêmica nacional.

A relevância de sua pesquisa transcende fronteiras, e sua presença como palestrante plenária no Congresso Nacional de Matemática Aplicada e Computacional proporcionaria uma oportunidade valiosa para os participantes terem acesso direto aos desenvolvimentos mais recentes e inovadores em teoria espectral dos grafos. Sua experiência como orientadora de alunos também destaca seu compromisso em inspirar e moldar a próxima geração de pesquisadores, o que faz dela não apenas uma líder em sua área, mas também uma mentora valiosa para jovens cientistas.

4. Palestras

As palestras serão divididas em dois blocos de duas horas cada. Abaixo o planejamento inicial das palestras:

Bloco 1:

Palestra 1 : Aida Monge Abiad, Eindhoven University of Technology.

Título : A unified framework for the Expander Mixing Lemma of irregular graphs and its applications.

Resumo : A unified framework of the Expander Mixing Lemma for irregular graphs using adjacency eigenvalues is presented, as well as several new versions of it. We also investigate some of its applications in graph theory, which include tight bounds on the zero forcing number and the routing number of a graph. To derive our results we use a new application of weight partitions of graphs, where the Perron eigenvector entries are used as vertex weights as a way to regularise the graph.

Palestra 2 : Cristiane Maria Sato, UFABC.

Título : A Primal-Dual Extension of the Goemans–Williamson Algorithm for the Weighted Fractional Cut-Covering Problem.

Resumo : We study a weighted generalization of the fractional cut-covering problem, which we relate to the maximum cut problem via antiblocker and gauge duality. This relationship allows us to introduce a semidefinite programming (SDP) relaxation whose solutions may be rounded into fractional cut covers by sampling via the random hyperplane technique. We then provide a $1/\alpha_{GW}$ -approximation algorithm for the weighted fractional cut-covering problem, where $\alpha_{GW} \approx 0.878$ is the approximation

factor of the celebrated Goemans–Williamson algorithm for the maximum cut problem. Nearly optimal solutions of the SDPs in our duality framework allow one to consider instances of the maximum cut and the fractional cut-covering problems as primal-dual pairs, where cuts and fractional cut covers simultaneously certify each other's approximation quality. We exploit this relationship to introduce new combinatorial certificates for both problems, as well as a randomized polynomial-time algorithm for producing such certificates. In particular, we show how the Goemans–Williamson algorithm implicitly approximates a weighted instance of the fractional cut-covering problem, and how our new algorithm explicitly approximates a weighted instance of the maximum cut problem. We conclude by discussing the role played by geometric representations of graphs in our results, and by proving our algorithms and analyses to be optimal in several aspects.

(This is joint work with N. Benedetto Proença, M.K. de Carli Silva, and L. Tunçel.)

Palestra 3 : Emanuel Juliano Morais Silva, UFMG.

Título : Grundy Spectral Bound.

Resumo : The Grundy number is the minimum number of colors needed to color a graph using the greedy algorithm, regardless of the ordering. Computing the Grundy number of a graph is an NP-Hard problem; however, there is a characterization in terms of induced subgraphs: a graph has a Grundy number of at least k if and only if it contains a k-atom. In this paper, we determine the smallest possible largest eigenvalue of a k-atom using properties of the matching polynomial. With this result, we present an upper bound for the Grundy number of a graph in terms of the largest eigenvalue of its adjacency matrix. Our bound is tight for an infinite family of graphs and provides an improvement on the known bounds for several sparse graphs.

(This is joint work with G. Coutinho.)

Palestra 4 : Frederico Cançado Pereira, UFMG.

Título : Common equitable partitions.

Resumo : In our work, we explore the symmetrized quotient graph and its connection to the theory of fractional isomorphism. The main focus is on establishing conditions for two graphs to share a common symmetrized quotient graph. These conditions are presented in the form of an original and innovative theorem: two graphs have a common symmetrized quotient if and only if there exists a specific non-negative matrix that satisfies certain properties related to their adjacency matrices. Additionally, a correspondence between equitable partitions and non-normalized quotients of graphs is established, highlighting how equitable partitions are inherited by quotient graphs. These results provide insights into the interaction between graph theory and matrix algebra, especially concerning graph quotients and equitable partitions.

(This is joint work with G. Coutinho.)

Palestra 5 : Chris Godsil, University of Waterloo.

Título : Tails of Graphs.

Resumo : We consider graphs obtained from a base graph X by joining one-sided infinite paths to a subset of the vertices of X. Both physicists and chemists are interested in the spectral properties of such graphs. I will present some of the relevant theory, discuss the appplications, and present some of the problems this work raises.

Bloco 2:

Palestra 1 : Krystal Guo, University of Amsterdam.

Título : Simplicial Laplacians of Hypergraphs.

Resumo : While the spectra of graphs is a well-established field of study, spectra of hypergraphs is a fairly new topic. There are several choices for how one defines the spectrum of a hypergraph; for example, one could take the adjacency matrix of the bipartite incidence graph between vertices and hyperedges, or of the tensor associated with the hypergraph. In this talk, we look at the combinatorial Laplacians and restrict our attention to hypergraphs arising from the clique (or co-clique) complexes of strongly regular graphs. We find the spectrum for some special families and give connections to the Möbius function of a poset motivation for this choice of matrix coming a model of supersymmetric quantum mechanics.

Palestra 2: Vilmar Trevisan, UFRGS.

Título : Recent developments on the Laplacian spectra of graphs with few distinct eigenvalues of the Distance and the Laplacian Distance matrices.

Resumo : A fundamental question in spectral graph theory concerns the *distribution* of the eigenvalues of a graph G. Given a graph G, its Laplacian matrix $L(G)$ has been extensively studied in the literature and is important in many areas of pure and applied science. In this talk we discuss some recent results on the spectral distribution of $L(G)$. We show, for example, that the number of Laplacian eigenvalues in $[0, 1)$ is bounded by the domination number. We illustrate how these findings led to advances in some open problems in spectral graph theory. In particular, for the classes of trees and threshold graphs, we find examples of equienergetic graphs, and determine the graph having largest Laplacian energy.

Palestra 3 : Marcel K. de Carli Silva, USP.

Título: Dual Hoffman Bounds for the Stability and Chromatic Numbers Based on SDP.

Resumo: The notion of duality is a key element in understanding the interplay between the stability and chromatic numbers of a graph. This notion is a central aspect in the celebrated theory of perfect graphs, and is further and deeply developed in the context of the Lovász theta function and its equivalent characterizations and variants. The main achievement of this paper is the introduction of a new family of norms, providing upper bounds for the stability number, that are obtained through duality from the norms motivated by Hoffman's lower bound for the chromatic number, and which achieve the (complementary) Lovász theta function at their optimum. As a consequence, our norms make it formal that Hoffman's bound for the chromatic number

and the Delsarte-Hoffman ratio bound for the stability number are indeed dual. Further, we show that our new bounds strengthen the convex quadratic bounds for the stability number studied by Luz and Schrijver, and which achieve the Lovász theta function at their optimum. One of the key observations regarding weighted versions of these bounds is that, for any upper bound for the stability number of a graph which is a positive definite monotone gauge function, its gauge dual is a lower bound on the fractional chromatic number, and conversely. Our presentation is elementary and accessible to a wide audience.

(This is joint work with N. Benedetto Proença and G. Coutinho.)

Palestra 4 : Luiz Emilio Allem, UFRGS.

Título : The minimum number of distinct eigenvalues of a threshold graph is at most 4.

Resumo : In this talk we present a constructive procedure that shows that the minimum number of distinct eigenvalues of a threshold graph is at most 4, $q(G) \leq 4$. More specifically, given a threshold graph G we construct a matrix $M \in S(G)$ such that the set of distinct eigenvalues of M, denoted by $\mathrm{DSpec}(M)$, satisfies $\mathrm{DSpec}(M) \subseteq \{-\lambda, 0, \lambda, 2\lambda\}$ for any $\lambda \in \mathbb{R}$.