Mini-symposium MS15, in the framework of CNMAC 2026

“Mathematical modelling of complex phenomena in fluid dynamics”
“Modelagem de Fenômenos Complexos em Dinâmica de Fluidos”

Organizers: Pavel Petrov (IMPA), Alexei Mailybaev (IMPA), Yulia Petrova (IME USP)

Room: Sala 5 - Bloco IV

Schedule:

	Tue, Sep 15 **Chair: Pavel Petrov**	**Wed, Sep 16** **Chair: Yulia Petrova**
08:00—08:24	**Prof. Dr. Eduardo Abreu (UNICAMP)** *“The no-flow Lagrangian-Eulerian approach in 3D for fluid dynamics and hyperbolic problems”*	**Dr. Erika Paola Ortiz Bernal (IMPA)** **“***Spontaneous stochasticity in the Landau–Lifshitz–Navier–Stokes equations on logarithmic lattices”*
08:24—08:48	**Dr. Hugo Tavares (LNCC)** *“Mathematical and numerical modeling of dispersed multiphase flows in scale-forming environments via mesoscopic methods”*	**Prof. Dr. Sergey Sergeev (UFRJ)** *“Semi-classical homogenization theory for linear gravity waves over a non-uniform bottom”*
08:48—09:12	**Marcio Marques (IMPA)** *Overcoming spectral bias and lack of causality in PINNs*	**Prof. Dr. Sergey Tikhomirov (PUC)** *Viscous and gravitational fingers in porous media*
09:12—09:36	**Dr. Bruno Magacho (IMPA)** *“Quantification of intermittency through hidden symmetry in 3D turbulence”*	**Prof. Dr. Maicon R. Correa (USP)** *“Sequential coupling strategies for black-oil flow in poroelastic media”*
09:36—10:00	**Prof. Dr. Fabio Pereira dos Santos (UFRJ)** *“Shearlet neural operators for anisotropic, shock-dominated, and multiscale parametric partial differential equations”*	**Prof. Dr. Pavel Petrov (IMPA)** *“Pseudodifferential mode parabolic equations and simulation of directional sound sources in underwater acoustics”*

ABSTRACTS

Spontaneous stochasticity in the Landau–Lifshitz–Navier–Stokes equations on logarithmic lattices

Erika Paola Ortiz Bernal *(presenting author)*
Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, Brazil
Email: erika.ortiz@impa.br

Co-authors:
Ciro Campolina (Scuola Normale Superiore, Pisa, Italy)
Alexei Mailybaev (Instituto de Matemática Pura e Aplicada – IMPA)

Abstract
The predictability of turbulent flows remains a central challenge in mathematics, physics, and geophysical sciences. In this work, we investigate the three-dimensional incompressible Landau–Lifshitz–Navier–Stokes equations formulated on logarithmic Fourier lattices with small-scale random forcing. Our focus is on the phenomenon of *spontaneous stochasticity*, whereby solutions remain stochastic in the limit of vanishing viscosity and noise.

To probe this behavior, we perform numerical simulations at increasing Reynolds numbers while simultaneously decreasing the noise amplitude. We analyze the statistics of individual large-scale Fourier modes and identify signatures of spontaneous stochasticity in two distinct scenarios: (i) evolution from rough initial data, and (ii) dynamics following a finite-time blowup of a strong solution. In both cases, convergence of probability density functions across different parameter regimes provides strong evidence that the limiting dynamics is described by a universal stochastic process.

These results are reported in a recent joint work.

References
E. Ortiz, C. S. Campolina, and A. A. Mailybaev, *Spontaneous stochasticity in the fluctuating Navier–Stokes equations on a logarithmic lattice*, to appear in *Philosophical Transactions of the Royal Society A* (2025). ArXiv: arXiv:2507.03196.

Keywords:
Spontaneous stochasticity; logarithmic lattices; vanishing noise

Quantification of intermittency through hidden symmetry in 3D turbulence

Bruno Magacho da Silva *(presenting author)*
Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, Brazil
Email: bruno.magacho@impa.br

Co-authors:
Simon Thalabard (Université Côte d'Azur, Nice, France)
Massimo Buzzicotti (University of Rome "Tor Vergata", Italy)
Francesco Bonaccorso (University of Rome "Tor Vergata", Italy)
Luca Biferale (University of Rome "Tor Vergata", Italy)
Alexei A. Mailybaev (IMPA, Brazil)

Abstract

Statistical hidden symmetries (HS) emerge from the Navier–Stokes equations in the limit of infinite Reynolds number when expressed in a quasi-Lagrangian reference frame and dynamically rescaled. However, these symmetries are extremely challenging to observe in direct numerical simulations (DNS) of the standard Navier–Stokes equations, since finite Reynolds number effects—particularly viscous dissipation—break scale invariance.

To overcome this limitation, we exploit the universality of inertial-range observables, which are expected to be largely independent of both forcing and dissipation mechanisms. Under this assumption, we consider an alternative dissipation model based on the Navier–Stokes–Smagorinsky (NSS) equations, where viscosity is replaced by a dynamical, scale-dependent, and Galilean-invariant effective viscosity.

Our results provide clear evidence of hidden symmetries through direct comparisons between Navier–Stokes and NSS simulations. Furthermore, we investigate intermittency by computing anomalous scaling exponents—deviating from Kolmogorov's 1941 theory—and by analyzing velocity structure functions. The results demonstrate that hidden symmetries are robustly observed across different resolutions within the NSS framework.

Reference

B. Magacho, S. Thalabard, M. Buzzicotti, F. Bonaccorso, L. Biferale, and A. A. Mailybaev, *Scale invariance of intermittency in LES turbulence*, Journal of Fluid Mechanics 1016, R5 (2025).

Keywords:

Turbulence; intermittency; hidden symmetry; Navier–Stokes; LES

Sequential coupling strategies for black-oil flow in poroelastic media

Maicon R. Correa *(presenting author)*
Department of Applied Mathematics, IME–USP, Brazil
Email: maicon@ime.usp.br

Co-author:
Marcio A. Murad (Department of Computational Modeling, LNCC, Brazil)

Abstract
In this talk, we discuss a Black-Oil model for flow in poroelastic media, where conservation laws are formulated in terms of component densities rather than the traditional volume-based equations at standard conditions. The resulting system describes the coupled dynamics of aqueous, oleic, and gaseous phases, incorporating additional complexities associated with transient porosity and formulated within a consistent Lagrangian framework.

This approach eliminates the need to explicitly track the hysteretic behavior of the bubble point pressure. Instead, phase appearance is treated through the evaluation of excess concentration relative to saturation limits in the oil phase.

We then develop and analyze two sequentially coupled fixed-stress algorithms, in which the flow, transport, and deformation subproblems are solved iteratively using appropriate numerical schemes. The performance and robustness of the proposed coupling strategies are demonstrated through numerical simulations of black-oil flow in poroelastic media.

References
J. A. Trangenstein and J. B. Bell, *Mathematical structure of the black-oil model for petroleum reservoir simulation*, SIAM Journal on Applied Mathematics 49(3), 749–783 (1989).
M. R. Correa and M. A. Murad, *A fixed-stress-split scheme for a black-oil model in poroelastic media*, Journal of Computational Physics 543, 114406 (2025).
M. R. Correa and M. A. Murad, *A new sequential method for three-phase immiscible flow in poroelastic media*, Journal of Computational Physics 373, 493–532 (2018).

Keywords:
Poroelasticity; black-oil model; sequential schemes; numerical methods for PDEs

Mathematical and numerical modeling of dispersed multiphase flows in scale-forming environments via mesoscopic methods

<u>Hugo S. Tavares</u> *(presenting author)*
Laboratório Nacional de Computação Científica (LNCC), Brazil
Email: hugoczpb@lncc.br

Co-authors:
M. Neumann (Universidade Federal do Rio de Janeiro – UFRJ, Brazil)
L. Moriconi (Universidade Federal do Rio de Janeiro – UFRJ, Brazil)
J. B. R. Loureiro (Universidade Federal do Rio de Janeiro – UFRJ, Brazil)

Abstract
This work addresses the numerical modeling of dispersed multiphase flows in scale-forming environments using mesoscopic simulation approaches. In particular, we investigate hybrid lattice Boltzmann formulations to simulate particle-laden flows and to model calcium carbonate (limescale) formation across a range of Reynolds numbers.

A key challenge in such systems is the accurate representation of flow dynamics in the presence of evolving fouling layers, which requires balancing physical realism with computational efficiency. Important factors include particle size, morphology, density, and chemical composition, all of which strongly influence both hydrodynamics and scaling kinetics. Since fully comprehensive models remain computationally prohibitive, we adopt tractable yet physically consistent modeling strategies.

Our approach combines a phase-field formulation for interfacial dynamics with an Eulerian–Eulerian framework for the transport of particles and ionic species. The resulting system is solved using a hybrid lattice Boltzmann method (LBM). Additionally, we perform comparisons with experimental data, employing data assimilation techniques based on Bayesian inference for model calibration.

Finally, we discuss alternative mesoscopic approaches—such as smoothed particle hydrodynamics (SPH) and multiphase particle-in-cell (MP-PIC) methods—as potential extensions toward the simulation of fully coupled multiphase regimes.

Keywords:
Dispersed multiphase flows; scale formation; mesoscopic methods; lattice Boltzmann

The no-flow Lagrangian–Eulerian approach in 3D for fluid dynamics and hyperbolic problems

Eduardo Abreu *(presenting author)*
Department of Applied Mathematics, University of Campinas (UNICAMP), Brazil
Email: eabreu@unicamp.br

Abstract
This work presents the construction and rigorous numerical analysis of a novel class of Lagrangian–Eulerian schemes for three-dimensional scalar and systems of hyperbolic conservation laws on structured and tetrahedral meshes. The approach is based on the mathematical framework of no-flow curves and manifolds, which reinterpret hyperbolic flux evolution as a vector field problem.

By establishing a connection with the theory of discontinuous ordinary differential equations with locally bounded cone variation, we provide a rigorous foundation for existence and continuous dependence, even in the presence of discontinuous, space–time dependent flux functions. This “no-flow” paradigm enables Riemann-solver-free computations, avoiding the need for spectral information or Jacobian evaluations, while naturally ensuring positivity and a weak CFL-type stability condition.

We present recent theoretical and numerical results for three-dimensional no-flow structures, extending the dual-step evolution–remap strategy to complex fluid dynamics problems, including the Navier–Stokes equations. Preliminary high-performance simulations of compressible Euler flows exhibiting Kelvin–Helmholtz instabilities are also discussed. In addition, multidimensional benchmarks—such as the Orszag–Tang MHD problem and non-strictly hyperbolic three-phase flows—confirm the consistency of the divergence structure and demonstrate the efficiency and scalability of the proposed framework.

References
E. Abreu et al., *A Jacobian-Free SDLE Framework for 3D Hyperbolic Systems*, submitted.
E. Abreu, E. Pandini, W. Lambert, *An Enhanced Lagrangian–Eulerian Method for a Class of Balance Laws*, Numerical Methods for PDEs 41 (2025).
E. Abreu et al., *A Lagrangian–Eulerian Method on Regular Triangular Grids for Hyperbolic Problems*, Journal of Dynamics and Differential Equations (2025).
E. Abreu et al., *Riemann problem solutions for a balance law under Dirac-Delta source*, Journal of Hyperbolic Differential Equations 21 (2024).
E. Abreu et al., *A class of Lagrangian–Eulerian shock-capturing schemes*, Journal of Scientific Computing 86 (2021).
Keywords: Hyperbolic conservation laws; Lagrangian–Eulerian methods; no-flow manifolds; numerical analysis; fluid dynamics

Viscous and gravitational fingers in porous media

<u>Sergey Tikhomirov</u> *(presenting author)*
University of São Paulo (IME-USP), Brazil
Email: sergey.tikhomirov@gmail.com

Co-authors:
F. Bakharev, A. Enin, S. Matveenko, D. Pavlov, N. Rastegaev, I. Starkov (St. Petersburg – Russia)
Ya. Efendiev (Texas A&M – USA);
K. Kalinin (Duke University – USA);
Y. Petrova (University of Sao Paulo – Brazil);

Abstract
The phenomenon of viscous fingering plays a central role in petroleum engineering, particularly in the displacement of highly viscous oils and in enhanced oil recovery techniques such as surfactant–polymer flooding. Experimental and numerical studies indicate a linear growth of the mixing zone; however, existing theoretical upper bounds for the growth rate exceed the velocities observed in simulations.

In this work, we present numerical strategies to determine the size of the mixing zone in both homogeneous and heterogeneous porous media, along with practical approaches to optimize injection schemes for enhanced oil recovery processes.

To improve theoretical estimates, we also investigate gravitational fingering, which arises from the unstable displacement of miscible fluids under gravity and is governed by Darcy's law. We propose that transverse convection plays a key role in slowing down finger growth and support this hypothesis through the analysis of a semi-discrete model.

We identify a characteristic structure of gravitational fingers, where propagation occurs through a sequence of traveling waves forming a "terrace" pattern. This structure provides new insight into the dynamics of fingering and its impact on transport processes in porous media.

References
F. Bakharev et al., *Velocity of viscous fingers in miscible displacement*, J. Comput. Appl. Math. 402 (2022).
F. Bakharev et al., *Optimal polymer slugs injection profiles*, J. Comput. Appl. Math. 425 (2023).
I. Starkov et al., *Non-monotonic growth rate of viscous fingers*, Comput Geosci 27 (2023).
F. Bakharev et al., *Velocity of viscous fingers: intermediate concentration*, J. Comput. Appl. Math. 451 (2024).
Y. Petrova et al, *Propagating terrace in gravitational fingering*, SIAM J. Math. Anal. 57 (2025).

Keywords: Porous media; viscous fingering; gravitational fingering; enhanced oil recovery;

Shearlet Neural Operators for Anisotropic, Shock-Dominated, and Multiscale Parametric Partial Differential Equations

Fabio Pereira dos Santos *(presenting author)*
Computational Modeling Coordination, Laboratório Nacional de Computação Científica (LNCC), Brazil
Email: fsantos@eq.ufrj.br

Co-authors:
Julio de Castro Vargas Fernandes (Coordination of Mathematical and Computational Methods**,** Laboratório Nacional de Computação Científica (LNCC), Brazil)

Abstract
Neural operators have emerged as powerful tools for learning solution operators of parametric partial differential equations (PDEs). Among existing architectures, Fourier Neural Operators (FNOs) rely on global Fourier representations, which are inherently nonlocal and may struggle to capture anisotropic structures, sharp gradients, and localized discontinuities typical of shock-dominated and multiscale regimes.

To address these limitations, we propose the Shearlet Neural Operator (SNO), an architecture based on shearlet frame decompositions. Shearlets provide directional, multiscale, and spatially localized atoms, offering near-optimal sparse approximations for functions with edges, fronts, and other singular structures. By learning operators in the shearlet domain and reconstructing predictions via the inverse transform, SNO combines spectral-style computational efficiency with enhanced locality and directional selectivity.

We assess SNO across six benchmark PDE families, including strongly anisotropic advection, heterogeneous diffusion, and nonlinear conservation laws with complex shock geometries—straight, curved, interacting, spiral, and polygonal fronts. Compared to FNO baselines, SNO consistently improves predictive accuracy and better resolves anisotropic and discontinuity-dominated features, demonstrating its suitability for multiscale, shock-dominated, and low-regularity solution spaces.

Overcoming Spectral Bias and Lack of Causality in PINNs

Marcio Marques *(presenting author)*
Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, Brazil
Email: marcio.marques@impa.br

Co-authors:
L. Mendonça (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
A. Bizzi (Imperial College London, United Kingdom)
L. Moreira (Universidade do Estado do Rio de Janeiro – UERJ, Brazil)
C. Oliveira (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
V. Balestro (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
L. Fernandez (Laboratório Nacional de Computação Científica – LNCC, Brazil)
D. Yukimura (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
P. Petrov (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
J. M. Pereira (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
T. Novello (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)
L. Nissenbaum (Instituto de Matemática Pura e Aplicada – IMPA, Brazil)

Abstract
Physics-Informed Neural Networks (PINNs) represent a versatile class of methods in scientific machine learning that offer mesh-free solutions, automatic differentiation, and a natural framework for inverse problems. However, they struggle with more complex problems due to spectral bias (poor learning of high frequencies) and lack of causality (incorrect temporal evolution). This is particularly challenging in wave equations, where information is highly localized, making the training likely to converge to trivial solutions. We propose two distinct solutions that tackle these problems from complementary perspectives: one focused on training, the other on architecture. On the training side, a stable adaptive training (SAT) modifies the optimization process by dynamically focusing on high-error regions, implicitly facilitating high-frequency learning and causal adherence. On the architecture side, Neural Spectral Architectures (NeuSA) redesign the network itself, projecting the PDE solution onto a spectral basis to eliminate spectral bias and integrating time via a Neural ODE to enforce causality by design. Both approaches demonstrate improved accuracy and convergence for wave problems and other PDEs, offering two strategies for building robust PINNs.

Keywords: PINNs, Spectral Bias, Lack of Causality, Adaptive Training, Neural Spectral Architectures

Semi-classical homogenization theory for linear gravity waves over a non-uniform bottom

Sergey Sergeev *(presenting author)*
Instituto de Matemática, Universidade Federal do Rio de Janeiro, Brazil
Email: sergeevse1@im.ufrj.br

Abstract
We consider the linear potential model for the propagation of surface water waves over long distances in a basin with a variable bottom. In particular, we focus on the case where the bottom contains rapidly oscillating components that significantly influence the profile of the propagating wave.

We address the problem of describing surface waves generated by smooth, localized initial data. To this end, we formulate an auxiliary homogenization problem: to construct an equation with smooth coefficients whose solution approximates that of the original system.

We reduce the original system of equations to an evolution equation posed on the free surface by means of the Dirichlet-to-Neumann operator. The resulting equation is governed by a pseudo-differential operator with rapidly oscillating coefficients reflecting the fine-scale structure of the bottom.

We then apply a homogenization procedure based on operator separation of variables (adiabatic approximation) in order to derive an effective equation with smooth coefficients. Finally, we apply semi-classical asymptotic analysis to obtain explicit asymptotic formulas for the solution of the Cauchy problem for the homogenized equation, describing the propagation of waves over a rapidly oscillating bottom.

Keywords: Surface waves, Non-uniform bottom, Localized initial data, Homogenization theory, Semi-classical analysis

Pseudodifferential mode parabolic equations and simulation of directional sound sources in underwater acoustics

Pavel Petrov *(presenting author)*
Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, Brazil
Email: pavel.petrov@impa.br

Co-authors:
Alena Zakharenko (Il'ichev Pacific Oceanological Institute, Vladivostok, Russia)

Abstract: The problem of sound propagation in a three-dimensional marine environment is considered in the framework of mode parabolic equation theory. Acoustic field for a time-harmonic source is represented in the form of a normal modes decomposition. The coefficients in this decomposition can be computed by means of numerical solution of mode parabolic equations. The directivity of the source within this context can be taken into account by the initial conditions of the latter equations. The transformation of the directivity function of the source into the Cauchy data is discussed. The developed theory is applied to the case of the marine vibrator simulation. The field produced by such vibrator in a 3D shallow-water waveguide is computed.

Keywords: Mode parabolic equations, normal modes, directional sources, marine vibrators